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BRIEF COMMUNICATION

APPLICATION OF TWO-FLUID ANALYSIS TO LAMINAR STRATIFIED OIL–WATER FLOWS

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1. INTRODUCTION

Taitel & Dukler (1976) proposed a two-fluid model for two-phase stratified gas–liquid flow in circular pipes. The complex geometry of stratified flow in a pipe prevents an exact analysis and this difficulty is resolved by using an equivalent diameter for the gas and liquid phases. The gas is treated as a flow in a closed channel bounded by the pipe walls and the gas–liquid interface; the liquid as a flow in an open channel bounded only by the pipe walls.

The objective of the present study was to investigate the application of a similar methodology to stratified oil–water flows which may be compared to the exact solution for 2-D flow (flow between infinitely wide flat plates) and a numerical solution for 3-D flow (flow in a circular pipe) for laminar flow of both fluids. It is found that where a solution of the Taitel & Dukler type predicts dependence of holdup on the Martinelli parameter only, the exact solution shows an additional dependence on the ratio of viscosity of the two phases. This can be important in oil–water flows, whereas in gas–liquid flows the viscosity ratio is very small and has little effect on the results.

2. APPROXIMATE MODEL

For gas–liquid flow in a circular pipe, by eliminating the pressure drop from the gas and liquid momentum balances, Taitel & Dukler derived:

$$X_{\mathrm{GL}}^2\left[(\tilde{u}_\mathrm{L}\tilde{D}_\mathrm{L})^{-n}\,\tilde{u}_\mathrm{L}^2\frac{\tilde{P}_\mathrm{L}}{\tilde{A}_\mathrm{L}}\right]-\left[(\tilde{u}_\mathrm{G}\tilde{D}_\mathrm{G})^{-m}\tilde{u}_\mathrm{G}^2\left(\frac{\tilde{P}_\mathrm{G}}{\tilde{A}_\mathrm{G}}+\frac{\tilde{P}_\mathrm{i}}{\tilde{A}_\mathrm{L}}+\frac{\tilde{P}_\mathrm{i}}{\tilde{A}_\mathrm{G}}\right)\right]=0, \quad [1]$$

where X_{GL}^2 is defined by

$$X_{\mathrm{GL}}^2=\frac{\dfrac{4C_\mathrm{L}}{D}\left(\dfrac{u_{\mathrm{Ls}}D\rho_\mathrm{L}}{\mu_\mathrm{L}}\right)^{-n}\dfrac{\rho_\mathrm{L}u_{\mathrm{Ls}}^2}{2}}{\dfrac{4C_\mathrm{G}}{D}\left(\dfrac{u_{\mathrm{Gs}}D\rho_\mathrm{G}}{\mu_\mathrm{G}}\right)^{-m}\dfrac{\rho_\mathrm{G}u_{\mathrm{Gs}}^2}{2}}. \quad [2]$$

Similar arguments may be used for liquid–liquid flows, and here we assume that the upper phase is the more viscous (oil) phase. Hence the behaviour of the two phases in the pipe is effectively the reverse of gas–liquid flow, leading to:

$$X_{\mathrm{OW}}^2\left[(\tilde{u}_\mathrm{W}\tilde{D}_\mathrm{W})^{-n}\tilde{u}_\mathrm{W}^2\left(\frac{\tilde{P}_\mathrm{W}}{\tilde{A}_\mathrm{W}}+\frac{\tilde{P}_\mathrm{i}}{\tilde{A}_\mathrm{W}}+\frac{\tilde{P}_\mathrm{i}}{\tilde{A}_\mathrm{O}}\right)\right]-\left[(\tilde{u}_\mathrm{O}\tilde{D}_\mathrm{O})^{-l}\tilde{u}_\mathrm{O}^2\frac{\tilde{P}_\mathrm{O}}{\tilde{A}_\mathrm{O}}\right]=0, \quad [3]$$

where X^2_{OW} is defined by

$$X^2_{OW} = \frac{\dfrac{4C_W}{D}\left(\dfrac{u_{Ws}D\rho_W}{\mu_W}\right)^{-n}\dfrac{\rho_W u^2_{Ws}}{2}}{\dfrac{4C_O}{D}\left(\dfrac{u_{Os}D\rho_O}{\mu_O}\right)^{-l}\dfrac{\rho_O u^2_{Os}}{2}}. \quad [4]$$

For a pipe flow, the dimensionless parameters will be identical to those defined by Taitel & Dukler (1976), except that the effective diameters of the two phases will be defined by

$$\tilde{D}_W = \frac{4\tilde{A}_W}{(\tilde{P}_W + \tilde{P}_i)} \quad \text{and} \quad \tilde{D}_O = \frac{4\tilde{A}_O}{\tilde{P}_O}.$$

For a 2-D flow, the dimensionless variables need to be calculated for the appropriate geometry, i.e.

$$\tilde{P}_L = w/H, \qquad \tilde{A}_L = hw/H^2, \qquad \tilde{u}_L = \tilde{A}/\tilde{A}_L = H/h,$$
$$\tilde{P}_G = w/H, \qquad \tilde{A}_G = (H-h)w/H^2, \qquad \tilde{u}_G = \tilde{A}/\tilde{A}_G = H/(H-h),$$
$$\tilde{P}_i = w/H,$$

where gas and liquid may be interchanged with oil and water, respectively. (Note that for 2-D flow the perimeters $\tilde{P}_L$, $\tilde{P}_G$ and $\tilde{P}_i$ are independent of the total height, unlike in the circular pipe case.)

3. TWO-DIMENSIONAL FLOWS

3.1. Approximate solution

For fully developed laminar flow *between* flat plates the friction factor is given by $f = 12/\mathrm{Re}$. For laminar flow *across* a flat plate, the effective distance is doubled. Thus, in gas–liquid flow the gas is a flow between flat plates with a separation $(H - \tilde{h})$ giving $\tilde{D}_G = (1 - \tilde{h})$ and the liquid is a flow across a flat plate of depth $\tilde{h}$ giving $\tilde{D}_L = 2\tilde{h}$.

The Martinelli parameter for laminar flow reduces to

$$X^2 = \frac{u_{Ls}\mu_L}{u_{Gs}\mu_G} = \frac{1}{\tilde{Q}\tilde{\mu}} \quad [5]$$

and [1] becomes

$$(1-\tilde{h})^3 - \left(\frac{2}{X^2_{GL}}\right)\tilde{h}^2(1+\tilde{h}) = 0. \quad [6]$$

Using similar arguments for oil–water flow, we find that the effective diameters are $\tilde{D}_O = 2(1 - \tilde{h})$ and $\tilde{D}_W = \tilde{h}$ giving, from [3]:

$$2X^2_{OW}(1-\tilde{h})^2(2-\tilde{h}) - \tilde{h}^3 = 0, \quad [7]$$

with the same Martinelli parameter as in [5]. The dependence of the height of the lower phase with the Martinelli parameter is shown in figure 1.

3.2. Exact solution

A solution for the height of the lower layer in laminar two-phase flow between flat plates can be obtained from the solution of the momentum balances for the two phases:

$$\mu_L\frac{d^2u_L}{dy^2} = \frac{dp}{dz} \quad [8]$$

and

$$\mu_G\frac{d^2u_G}{dy^2} = \frac{dp}{dz}, \quad [9]$$

subject to the boundary conditions of no slip at the walls, and continuity of velocity and shear stress at the interface. This was considered by Denn (1980) and Russell & Charles (1959), and the height of the lower phase is given by

$$\tilde{h}^4[(1-\tilde{\mu})(\tilde{\mu}\tilde{Q}+1)] + 2\tilde{h}^3[\tilde{\mu}(\tilde{Q}+3) - 2] - 3\tilde{h}^2[\tilde{\mu}(\tilde{Q}+3) - 2] + 4\tilde{h}(\tilde{\mu}-1) + 1 = 0. \quad [10]$$

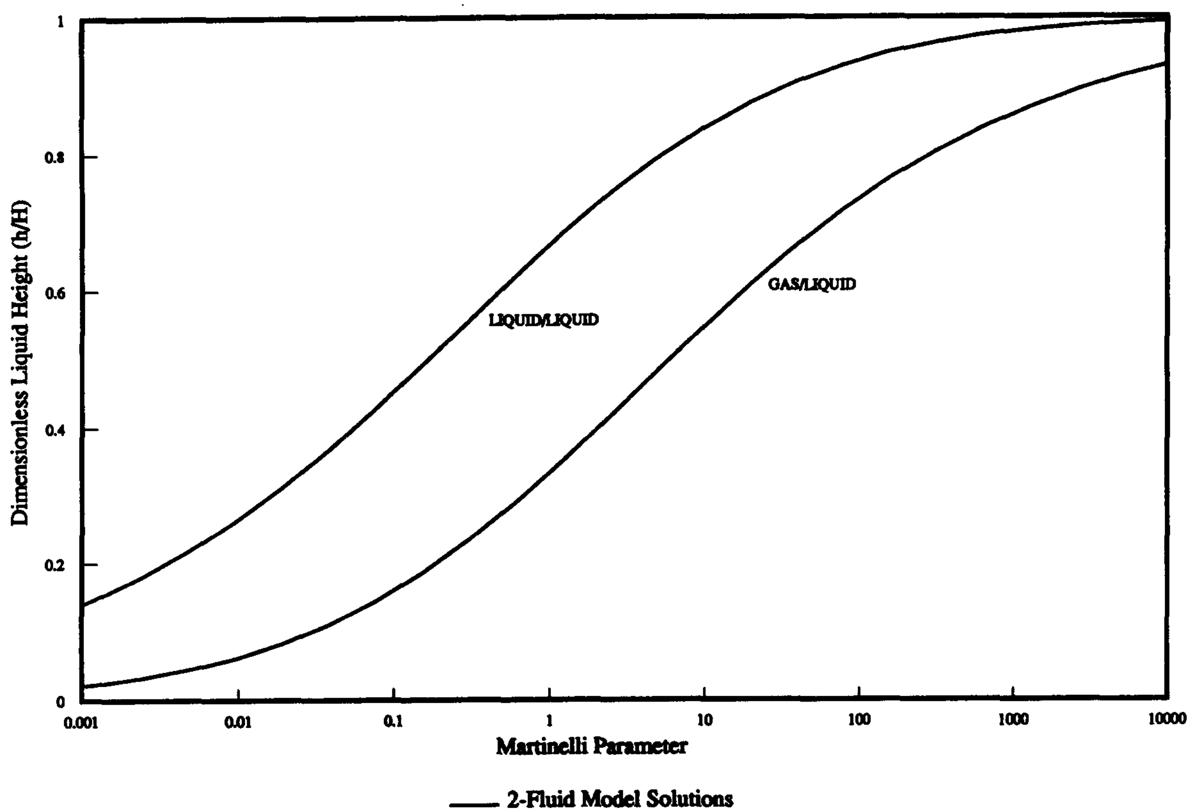

Figure 1. Two-phase flow between flat plates (two-fluid model calculations).

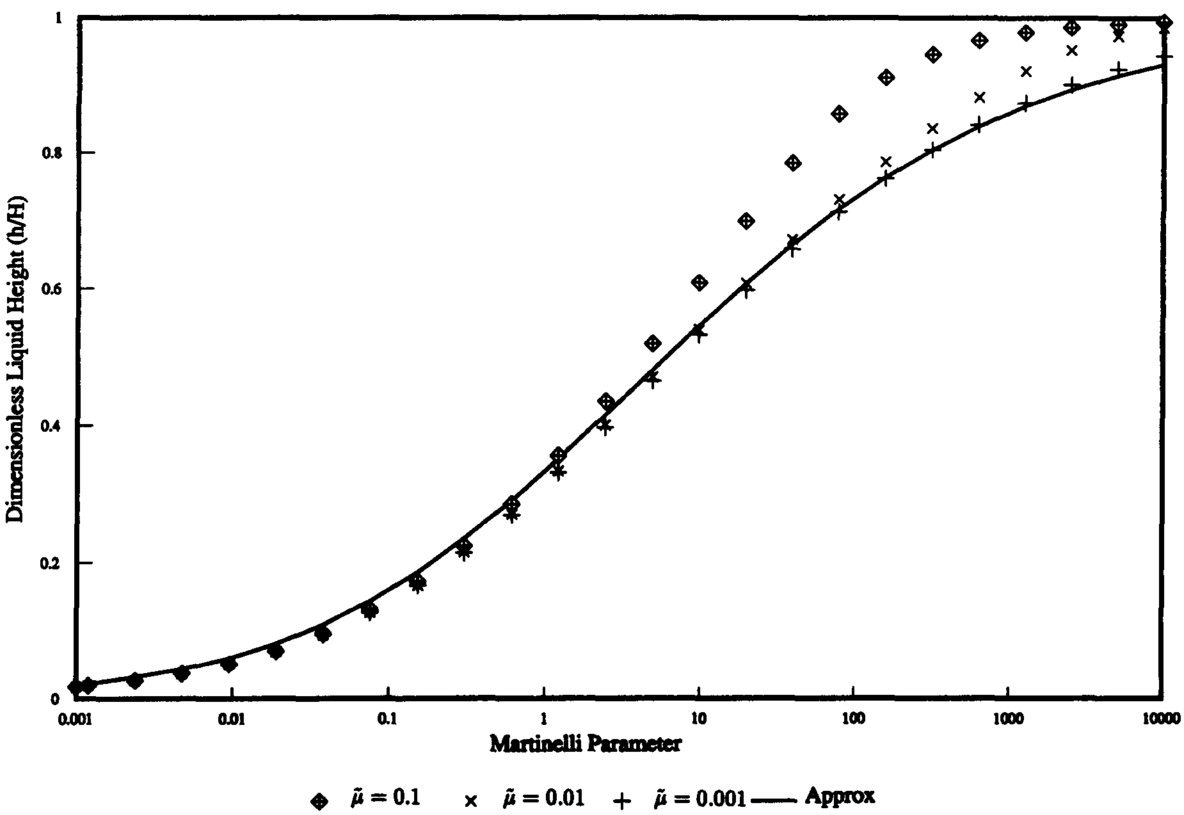

Figure 2. Gas–liquid flow between flat plates (comparison of exact solutions with the two-fluid model).

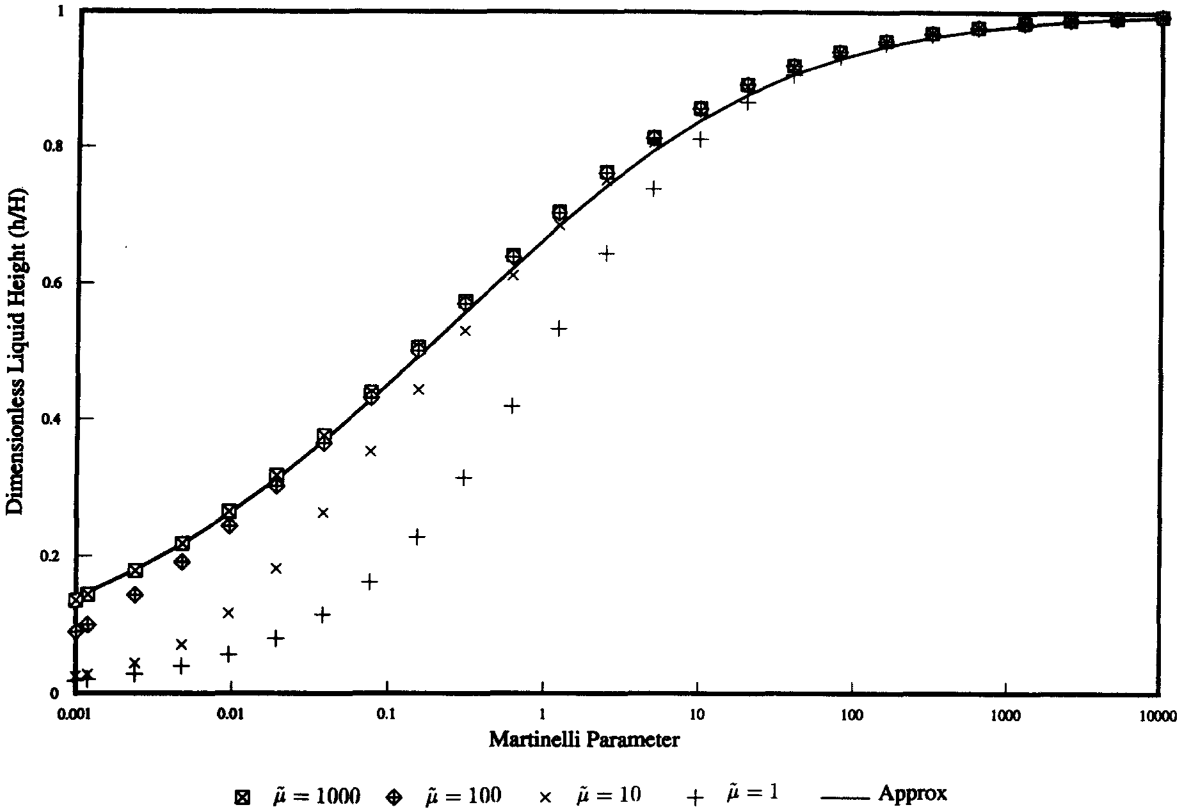

Figure 3. Liquid–liquid flow between flat plates (comparison of exact solutions with the two-fluid model).

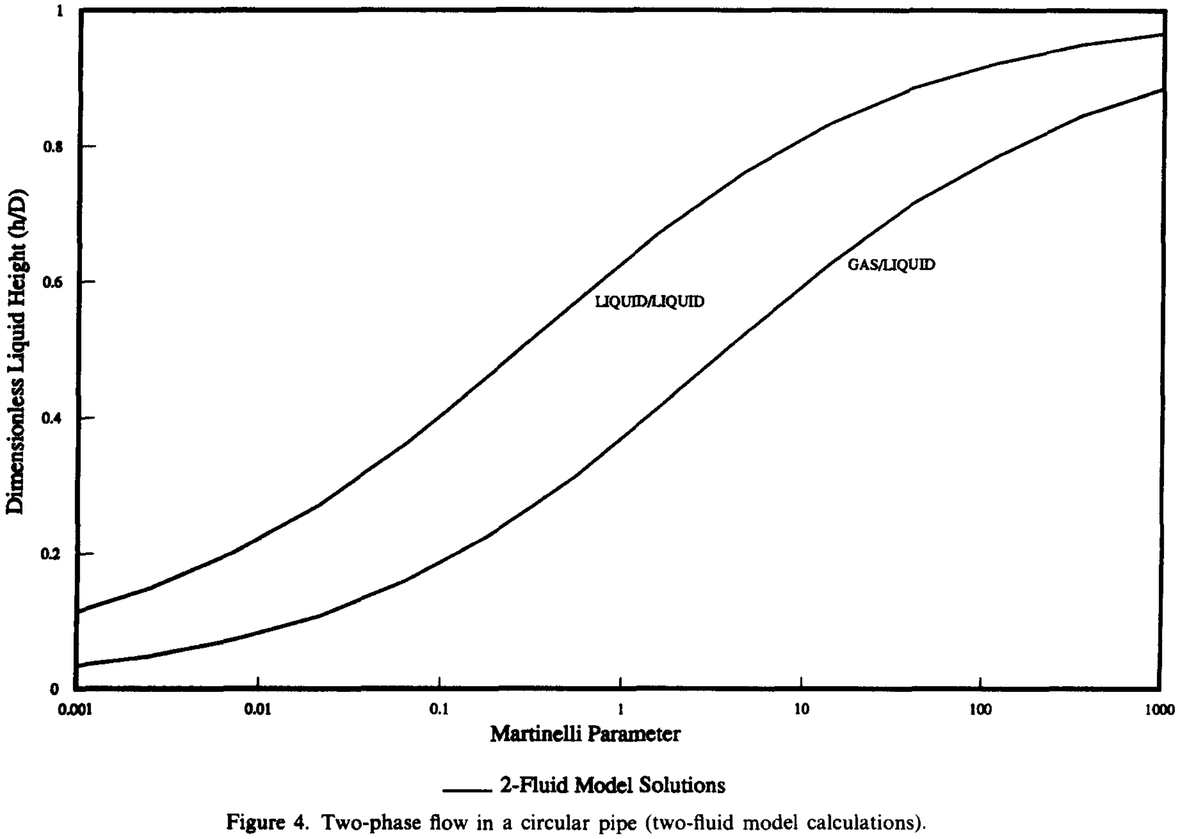

Figure 4. Two-phase flow in a circular pipe (two-fluid model calculations).

Note that $\tilde{\mu}$ and $\tilde{Q}$ do not always appear as the product $\tilde{\mu}\tilde{Q}$. Therefore, $\tilde{h}$ will vary with both the Martinelli parameter, $1/\tilde{\mu}\tilde{Q}$, and also the viscosity ratio, $\tilde{\mu}$.

The variation of the lower phase height with both the viscosity ratio and Martinelli parameter may therefore be easily calculated and compared to the solution given by the two-fluid model. This is shown in figure 2 for gas–liquid flow and in figure 3 for liquid–liquid flow. For gas–liquid flows, a significant deviation from the two–fluid solution is only observed at viscosity ratios above about 0.1, which are unlikely to occur in reality. Thus, for gas–liquid flow the two–fluid solution represents the real behaviour well. For liquid–liquid flows, however, significant deviations are observed for viscosity ratios in the region of 10, which are observed in practice.

4. THREE-DIMENSIONAL FLOWS

The solution of the approximate equations [1] and [3] is shown in figure 4 for laminar two-phase (gas–liquid and liquid–liquid) flows in a circular pipe. In the circular pipe geometry, the exact solution is obtained from the momentum conservation equations

$$\frac{d^2u_L}{dx^2}+\frac{d^2u_L}{dy^2}=\frac{1}{\mu_L}\frac{dp}{dz} \qquad [11]$$

and

$$\frac{d^2u_G}{dx^2}+\frac{d^2u_G}{dy^2}=\frac{1}{\mu_G}\frac{dp}{dz}. \qquad [12]$$

An analytical solution is not practically possible, and it is therefore necessary to resort to a numerical technique. For stratified two-phase flow in pipes, a numerical solution using bipolar coordinates is possible, as described by Issa (1988). The flow is assumed to be steady and

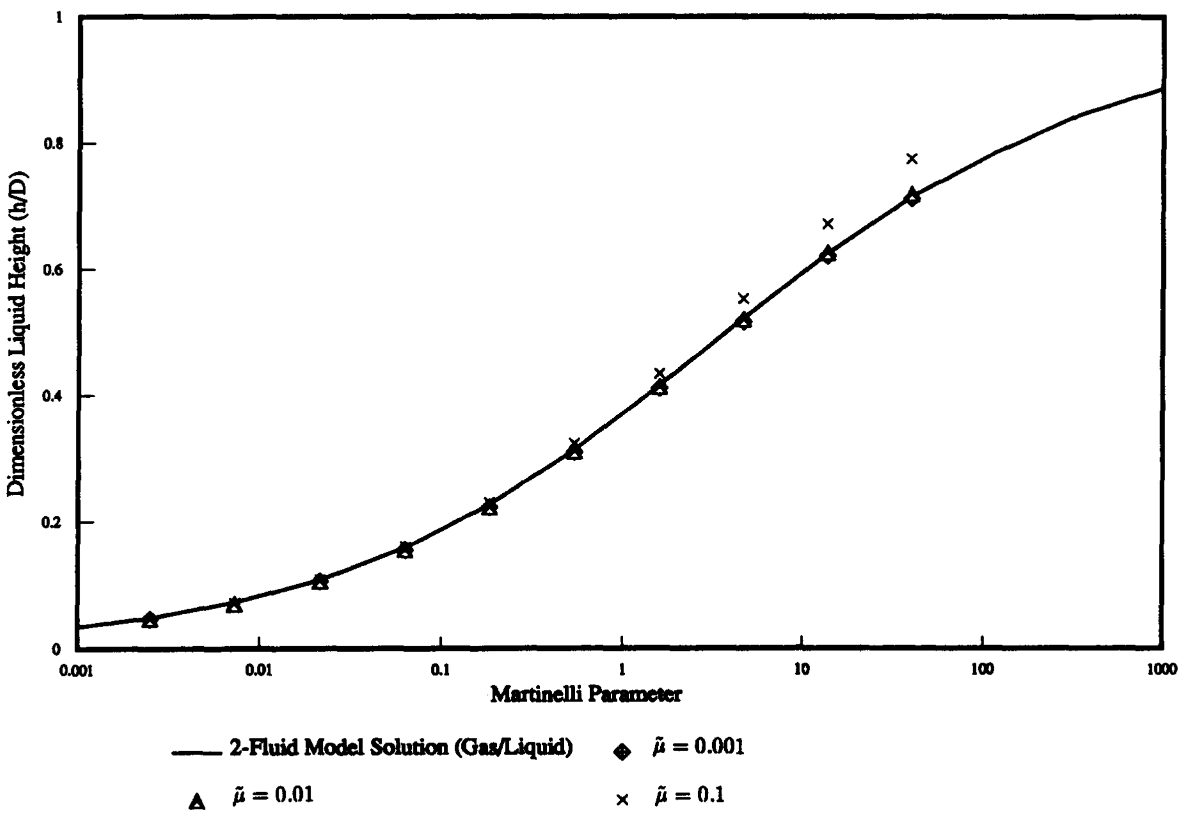

Figure 5. Gas–liquid flow in a circular pipe (comparison of numerical calculations with the two-fluid model).

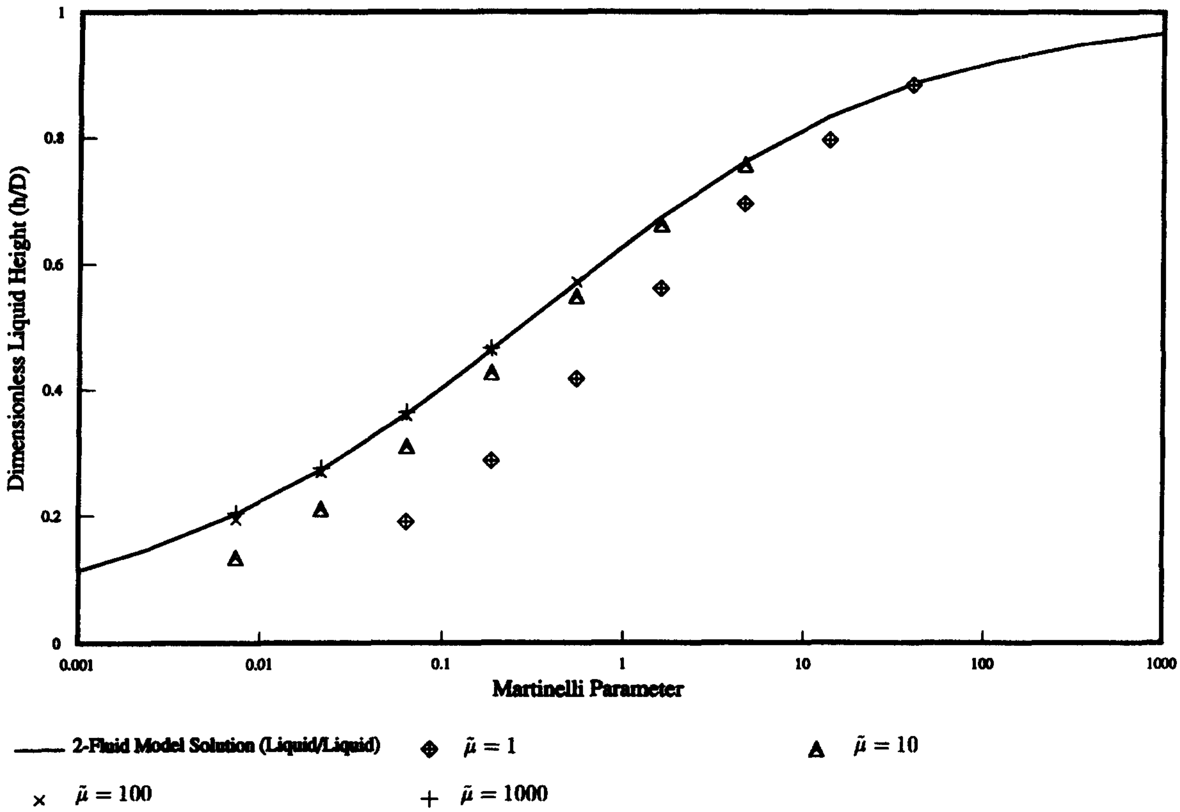

Figure 6. Liquid–liquid flow in a circular pipe (comparison of numerical calculations with the two-fluid model).

fully-developed with both phases in laminar flow, with a smooth interface. The boundary conditions of no slip at the walls and continuity of velocity and shear stress at the interface are once again assumed. A bipolar coordinate grid has the advantage of matching both the pipe walls and the interface in this flow configuration, and thus the boundary conditions can be applied at exact grid locations.

The variation of the lower phase height with the Martinelli parameter and viscosity ratio can be calculated relatively easily by this means, giving the results shown in figures 5 and 6. For gas–liquid flow, the agreement between the numerical results and the two–fluid model is good, even up to a viscosity ratio of 0.1, which agrees more closely than in the 2-D case. For liquid–liquid flows the same behaviour is observed, with significant deviation from the two-fluid model once the viscosity ratio falls below about 10.

5. CONCLUSIONS

An approximate model based on the Taitel & Dukler (1976) analysis for the holdup in flows between flat plates and flows in circular pipes was derived for gas–liquid and liquid–liquid flows. For laminar 2-D flows, an analytical solution may be obtained, while for 3-D flows, a numerical solution is required. For all physically realistic gas–liquid flows, the agreement between the two–fluid models and the exact solution in both 2-D and 3-D geometry is very close. For oil–water flows where the viscosity ratio can vary from approx. 1 upwards, there is a significant deviation in holdup prediction for lower viscosity ratios. This deviation is smaller, however, in a circular pipe geometry than in the 2-D flow case.

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